Graph topology and habitat assortativity drive phenotypic differentiation in an eco-evolutionary model

Victor Boussange & Loic Pellissier

CCS2021

Large-scale geographical patterns of species diversity

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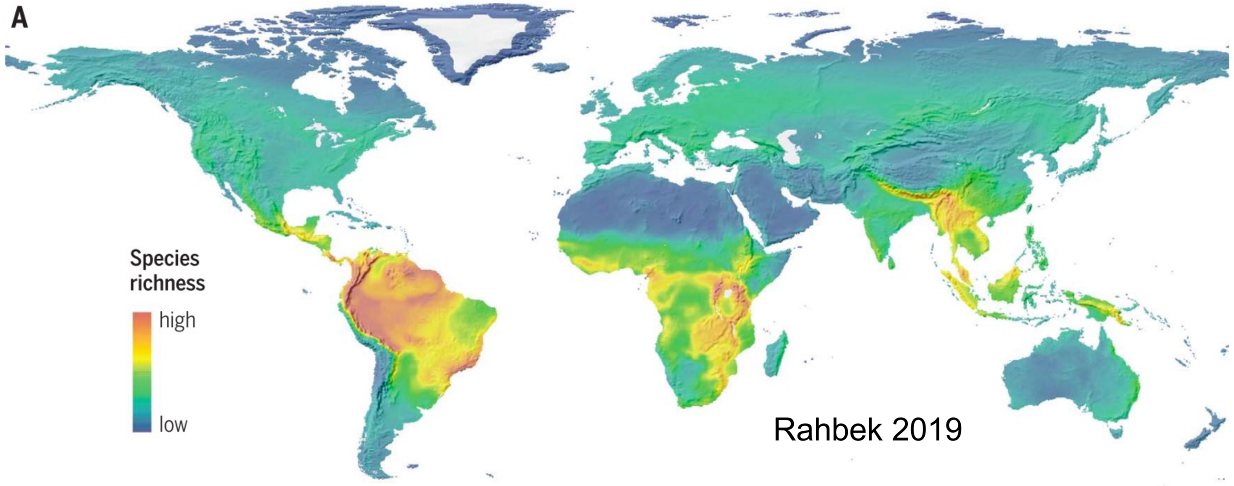
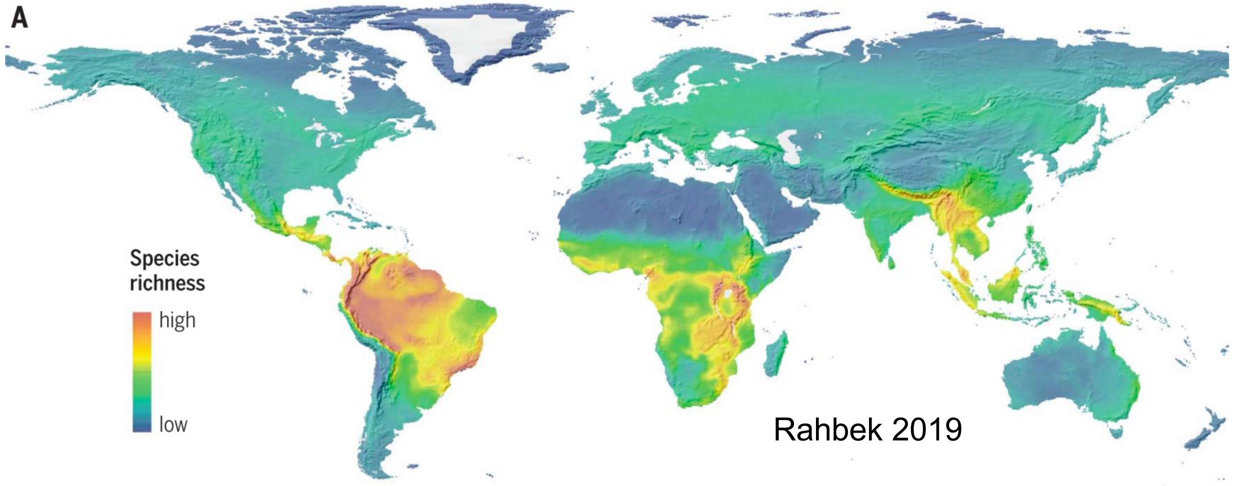


Rahbek 2019

Large-scale geographical patterns of species diversity



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Mountains represent **25 % of land area, but 85% of the world’s species** of amphibians, birds and mammals, many entirely restricted to mountains (Rahbek 2019)

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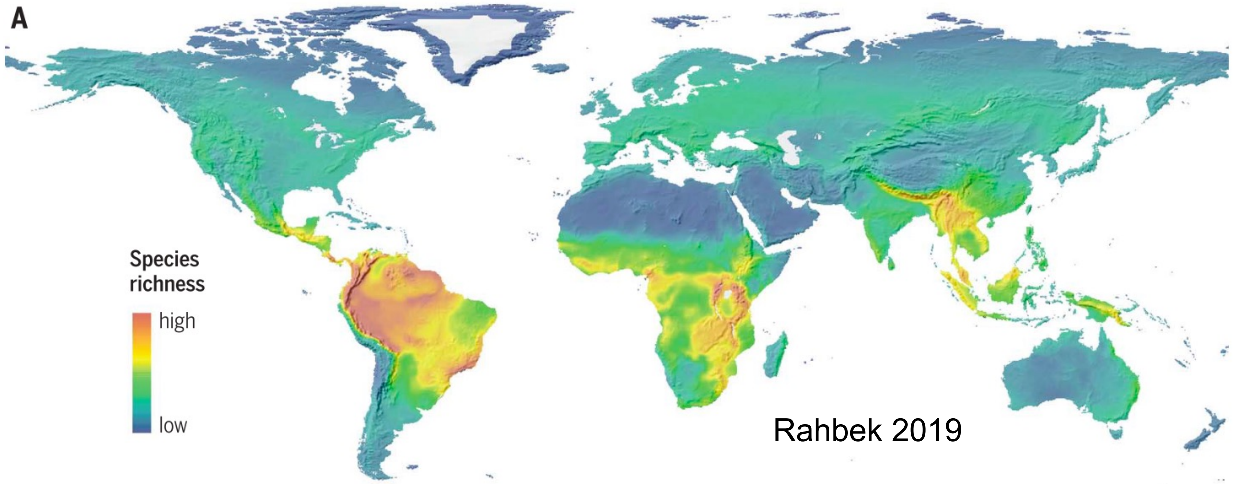

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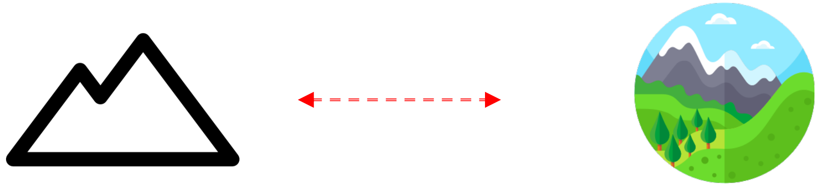
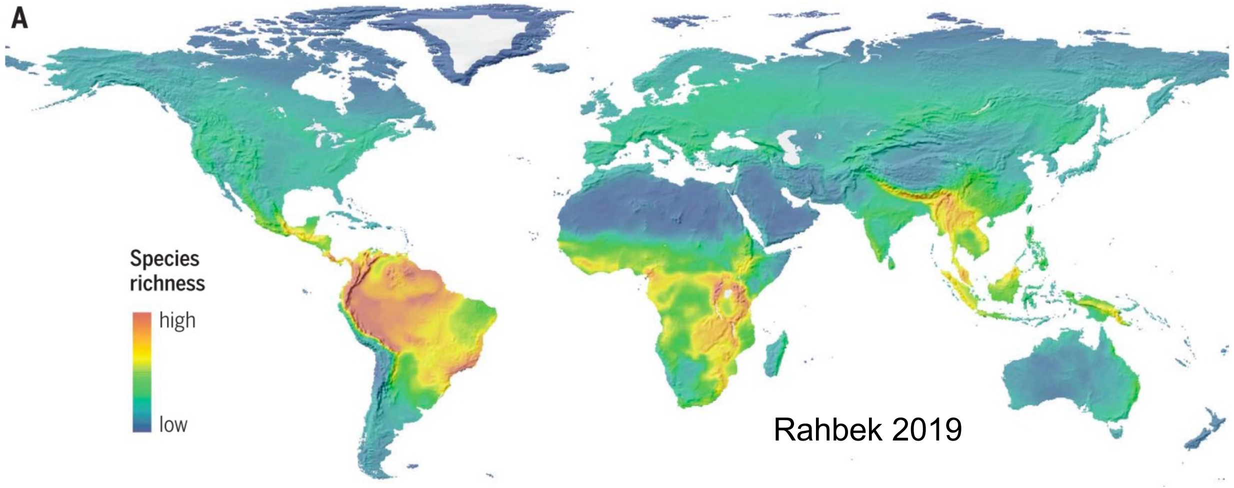


Topological constraints **Habitat heterogeneity**

Large-scale geographical patterns of species diversity



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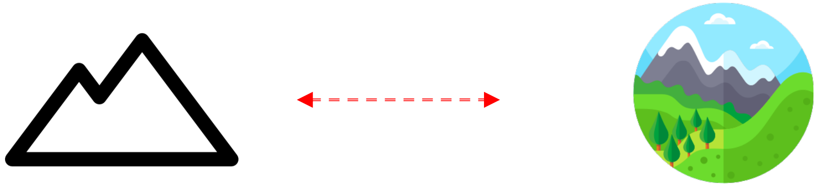
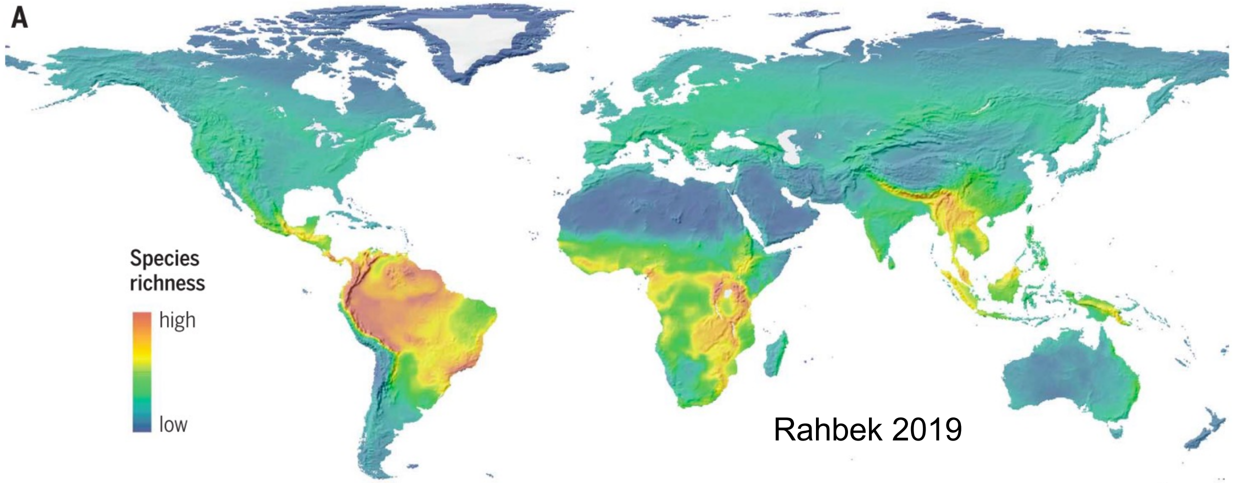


Topological constraints **Habitat heterogeneity**

Underlying processes?

Large-scale geographical patterns of species diversity



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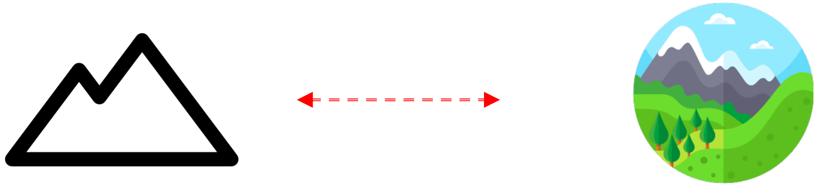

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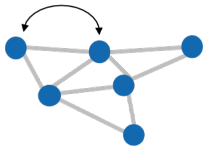
First principles modelling approach



Graph representation of the landscape

$$m\partial_t v = \sum_i F_i$$

Eco-evolutionary individual based model

Graphs as landscape abstraction

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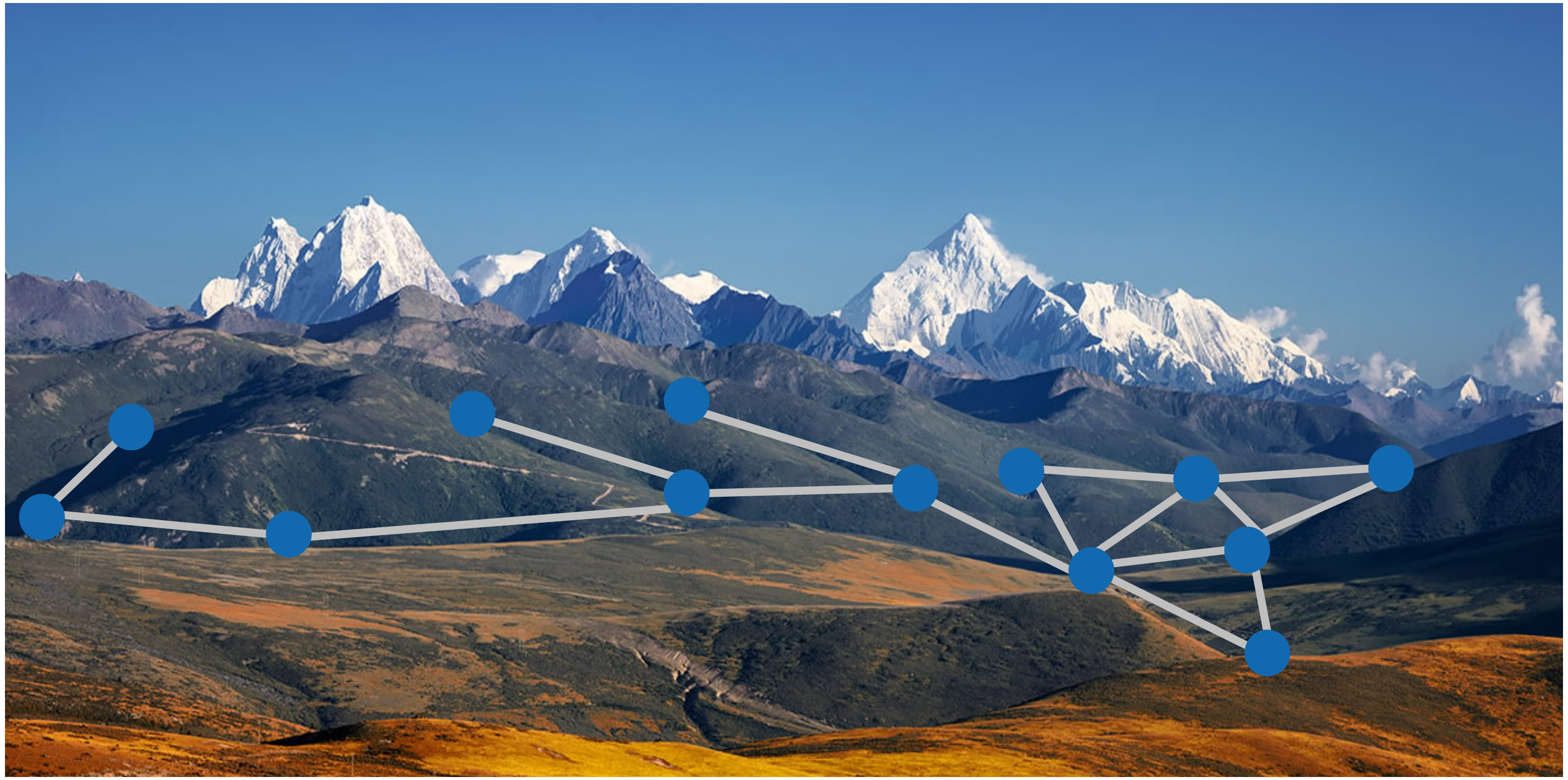

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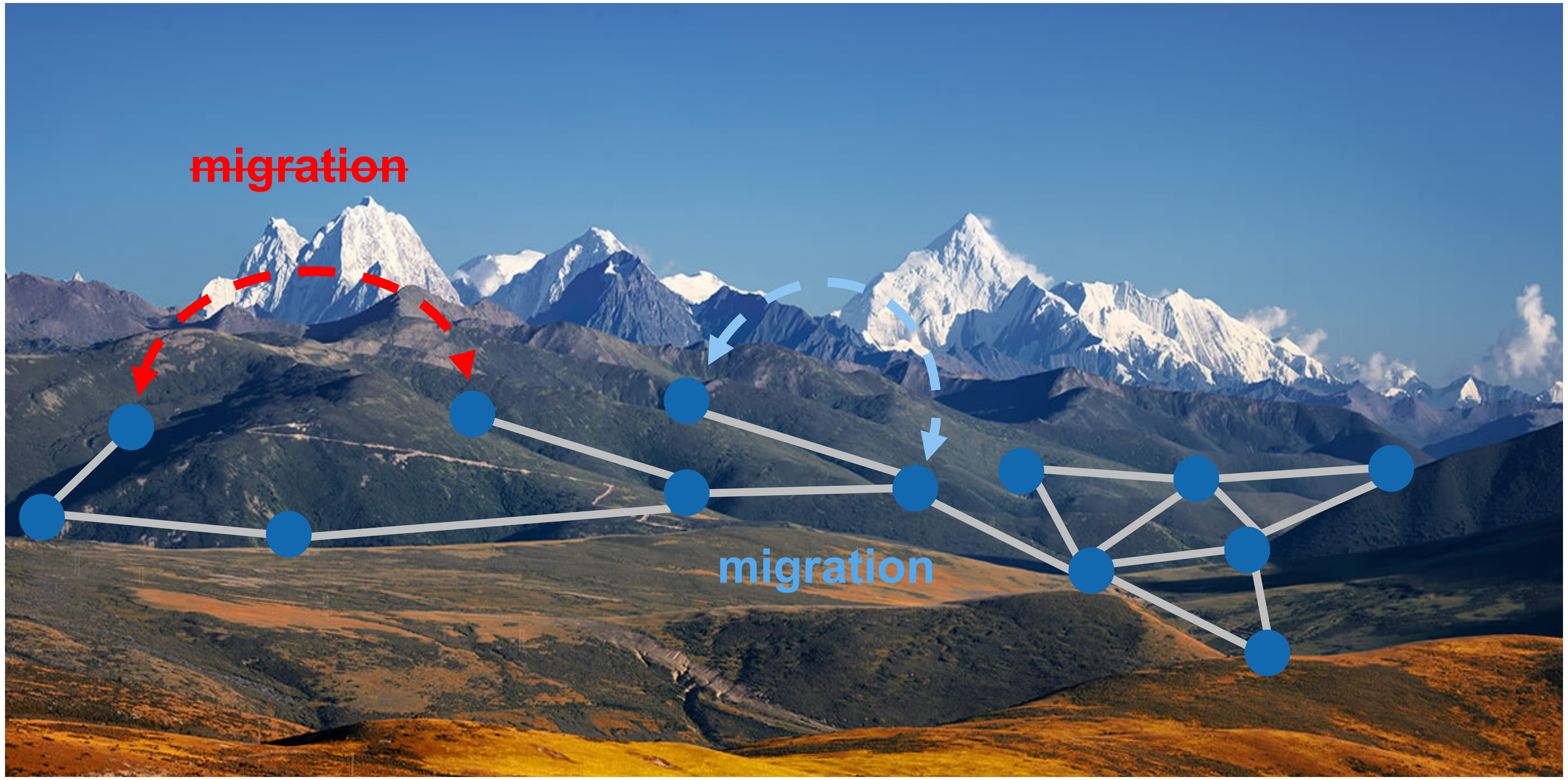
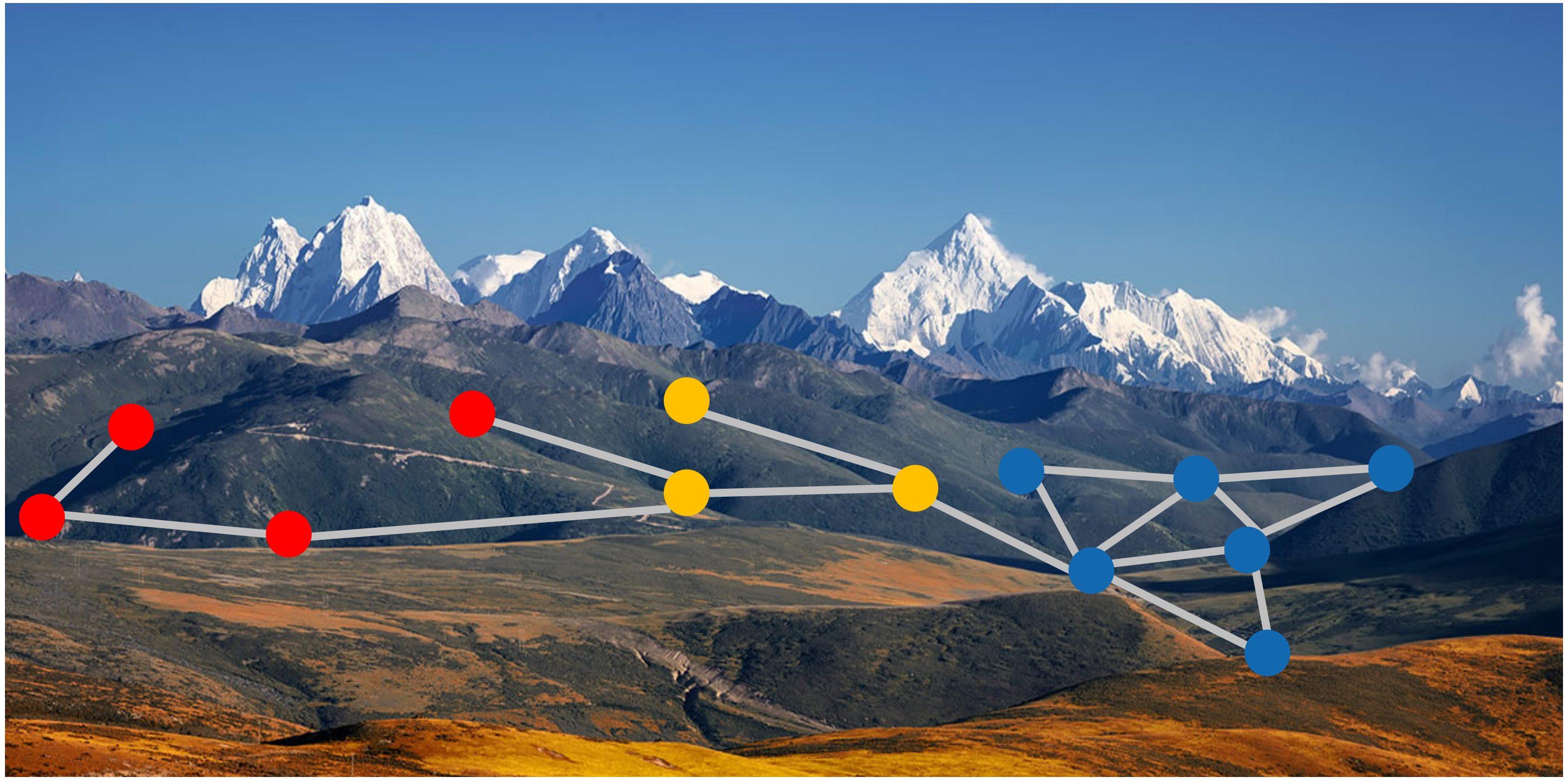


Graphs, to capture dispersal patterns

➔ Topological constraints

Graphs as landscape abstraction



and **environmental heterogeneity**

🔴 Habitat 1

🟡 Habitat 2

🔵 Habitat 3

Eco-evolutionary model

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Champagnat et al. 2006

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Champagnat et al. 2006

- **Measure-valued point process**

Eco-evolutionary model

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- **Measure-valued point process**
  - Individuals are represented by **dirac functions**

$\delta_{x_k^{(i)}}$ ← traits of individual k on vertex V_i

Eco-evolutionary model

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$$\delta_{x_k^{(i)}}$$

traits of individual k on vertex V_i

$$\nu^{(i)} = \sum_k^{N^{(i)}} \delta_{x_k^{(i)}}$$

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Ecology 🌱

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- Individuals **die** **Ecology** 🌱
  - competition for a finite amount of ressource

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Population size on V_i

$$d_i(x_k) \equiv \frac{N^{(i)}}{K}$$

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$$b_i(x_k)$$

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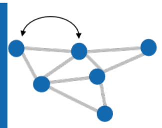

- Offsprings can migrate to neighboring vertices

Dispersal

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$$b_i(x_k)$$

$$m$$

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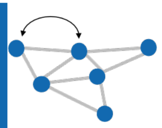

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Dispersal

- Offsprings' traits slightly differ from their parents'
  - Difference follows ~ $\mathcal{N}_{0,\sigma_\mu}$

Evolution

$\delta_{x_k^{(i)}}$

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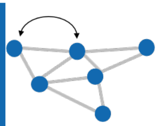

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Evolution

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Rates

Population size on V_i

$$d_i(x_k) \equiv \frac{N^{(i)}}{K}$$

$$b_i(x_k)$$

$$m$$

$$\mu$$

Expected dynamics

- Expected time variation of the process $L\phi(\nu_t^{(i)}) = \partial_t \mathbb{E}\Big[\phi(\nu_t^{(i)})\Big]$

$$
\begin{aligned}
L\phi(\nu_t^{(i)}) = & \int_{\mathcal{X}} \Big\{ b_i(\mathbf{x})(1-\mu)(1-m)(\phi(\nu_t^{(i)} + \delta_{\mathbf{x}}) - \phi(\nu_t^{(i)})) \Big\} \nu_t^{(i)}(d\mathbf{x}) && \text{births w/o mutations, w/o migrations} \\
& + \int_{\mathcal{X}} \Big\{ \mu(1-m) \int_{\mathcal{X}} b_i(y)(\phi(\nu_t^{(i)} + \delta_z) - \phi(\nu_t^{(i)}))\mathcal{M}(\mathbf{x}, y) dy \Big\} \nu_t^{(i)}(d\mathbf{x}) && \text{births w/ mutations, w/o migrations} \\
& + \iint_{\mathcal{X}} \Big\{ \frac{1}{K}(\phi(\nu_t^{(i)} - \delta_{\mathbf{x}})) - \phi(\nu_t^{(i)}))\nu_t^{(i)}(dy)\, \nu_t^{(i)}(dx) \Big\} && \text{deaths} \\
& + \sum_{j \neq i} \frac{a_{i,j}}{d_j} \int_{\mathcal{X}} \mu m \Big\{ \int_{\mathcal{X}} b_j(y)(\phi(\nu^{(j)} + \delta_{\mathbf{x}}) - \phi(\nu^{(j)}))\mathcal{M}(\mathbf{x}, y) dy \Big\} \nu_t^{(j)}(d\mathbf{x}) && \text{migrations w/ mutations} \\
& + \sum_{j \neq i} \frac{a_{i,j}}{d_j} \int_{\mathcal{X}} \Big\{ b_j(\mathbf{x})(1-\mu)m(\phi(\nu^{(j)} + \delta_{\mathbf{x}}) - \phi(\nu^{(j)})) \Big\} \nu_t^{(j)}(d\mathbf{x}). && \text{migrations w/o mutations}
\end{aligned}
$$

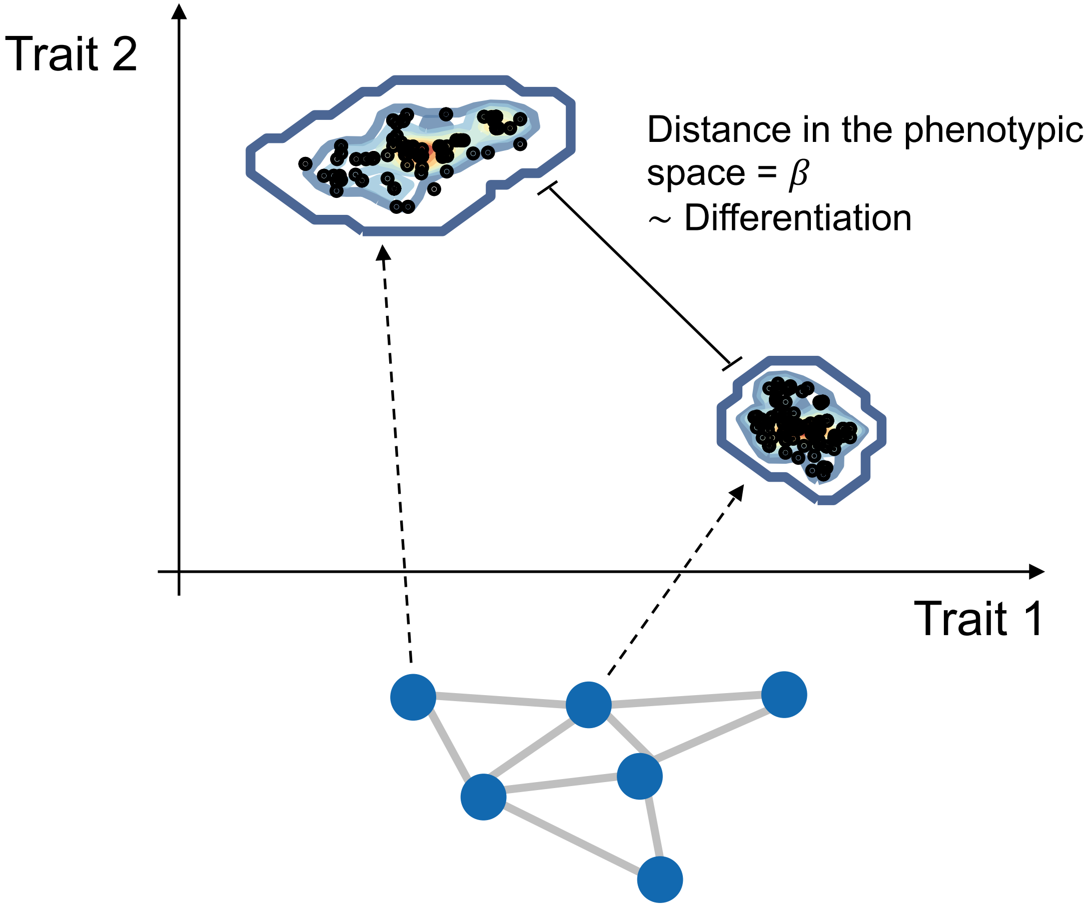
LANDSCAPE ECOLOGY

Differentiation

Variance of the mean trait $\bar{x}^{(i)}$ across nodes

$$\beta = \frac{1}{2M} \sum_i \sum_j \left(\bar{x}^{(i)} - \bar{x}^{(j)}\right)^2$$

number of vertices



Distance in the phenotypic space = β
~ Differentiation

Setting #1 – Effect of topology on differentiation

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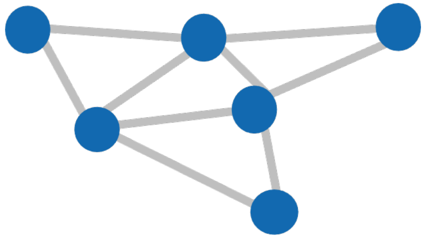

$$b_i(x_k) \equiv b \in \mathbb{R}$$

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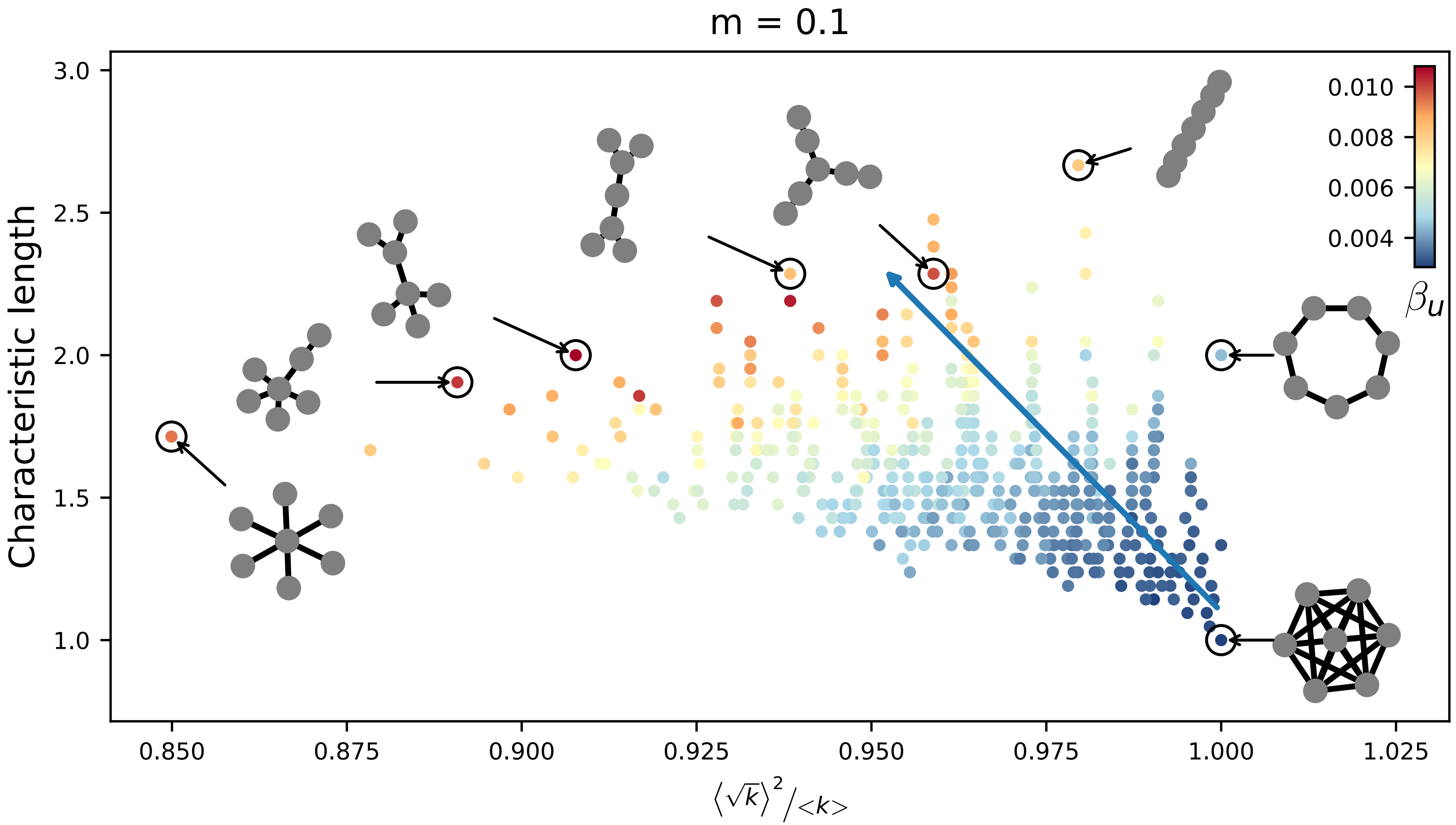
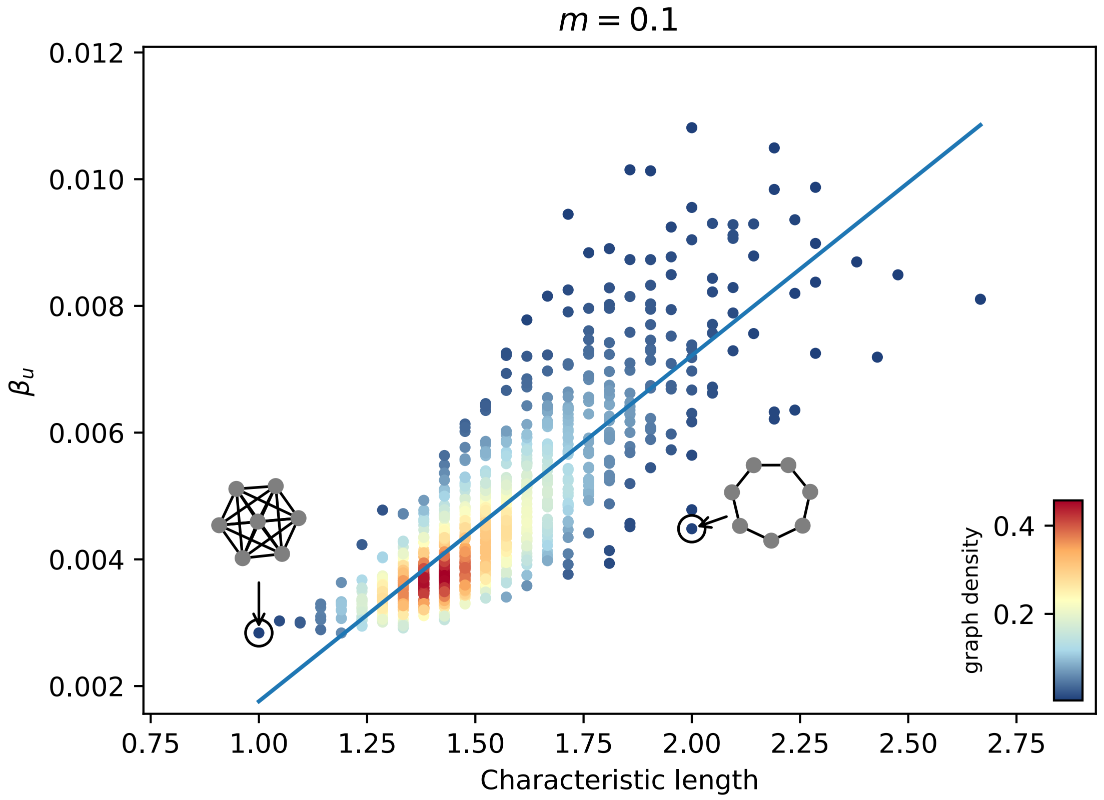


Setting #1 – Effect of characteristic length on differentiation



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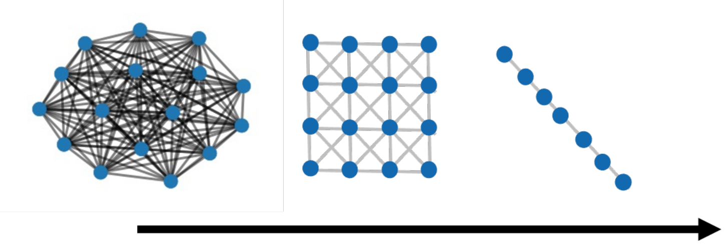
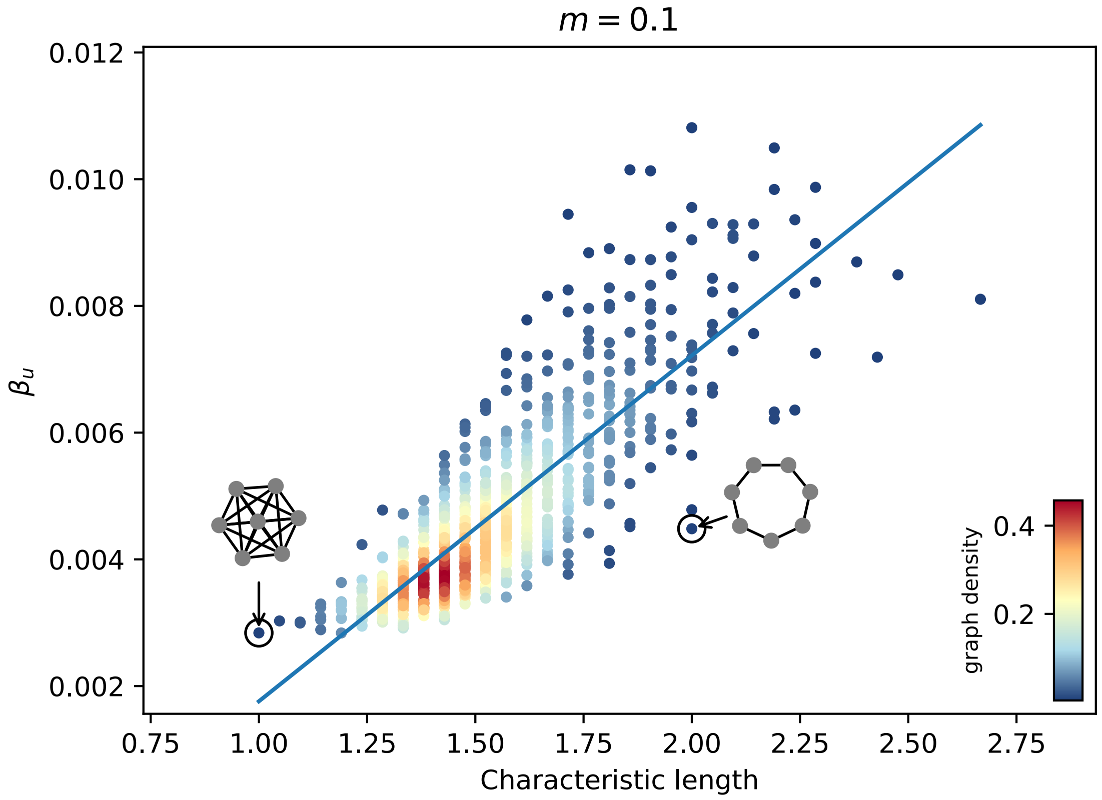
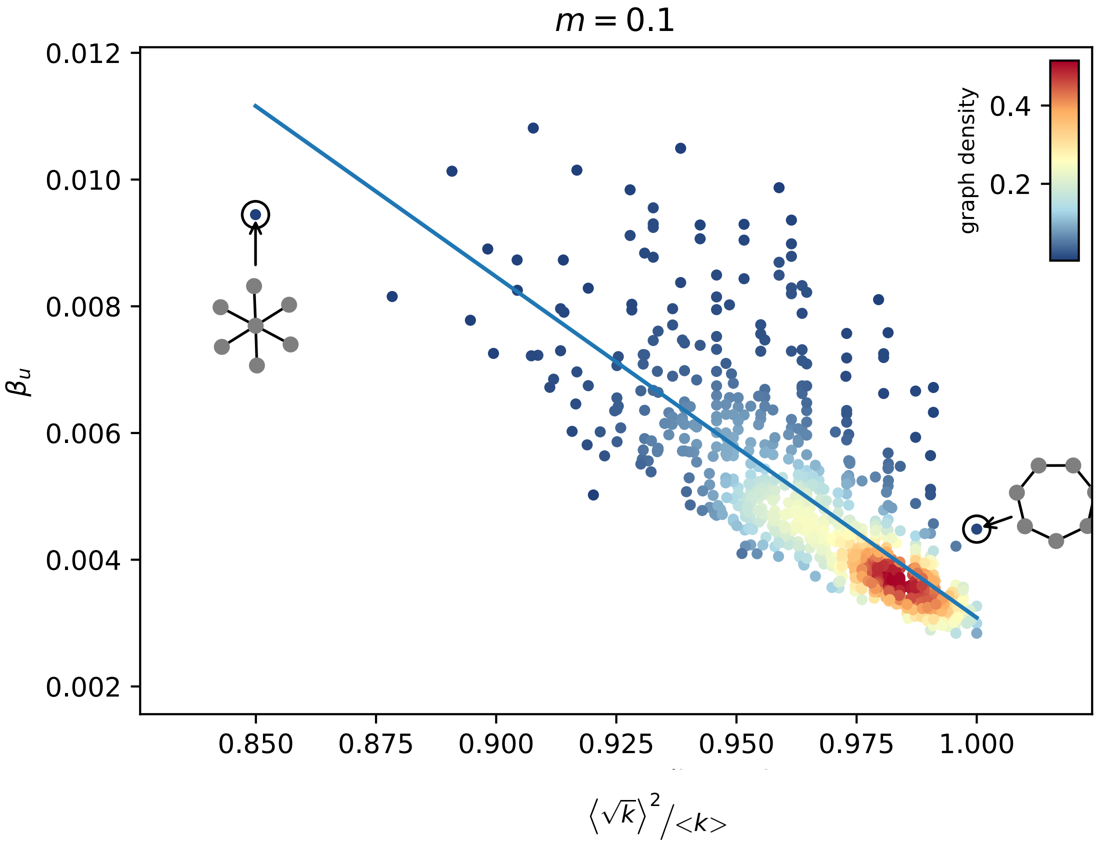


Characteristic length
~ landscape dimensionality



Setting #1 – Effect of heterogeneity in degree on differentiation



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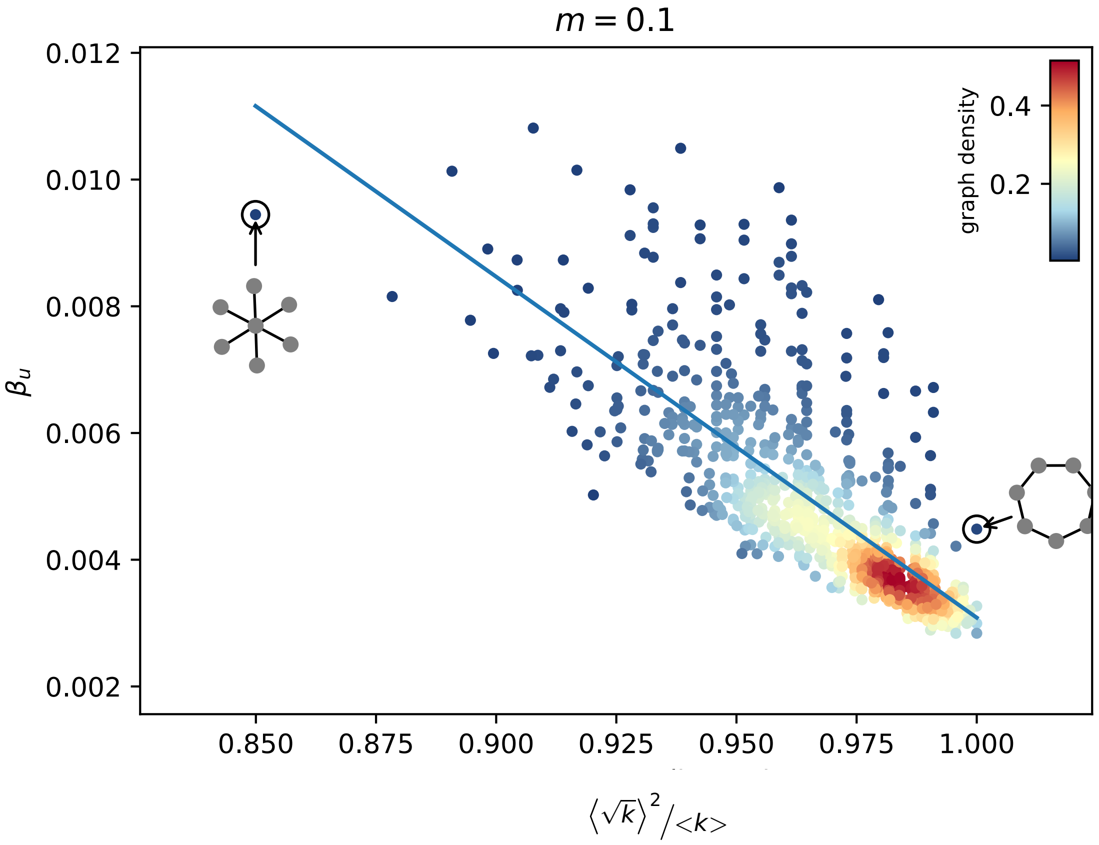


$$\partial_t N_t^{(i)} = N_t^{(i)}\left[b(1-m) - \frac{N_t^{(i)}}{K}\right]$$

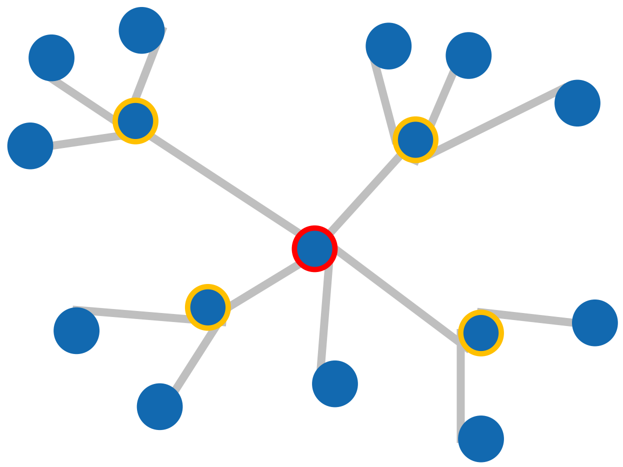
$$+ mb\sum_{j\neq i}\frac{a_{i,j}}{d_j}N_t^{(j)}$$

Mean field approach: all vertices having the same degree are equivalent

$$\bar{N} = bK\frac{\langle\sqrt{k}\rangle^2}{\langle k\rangle}$$

Average degree of the graph

Setting #1 – Effect of heterogeneity in degree on differentiation



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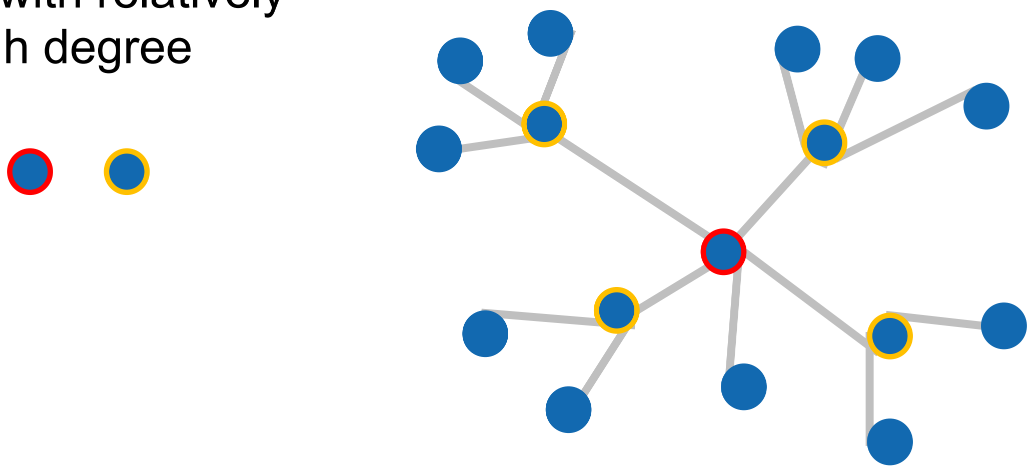


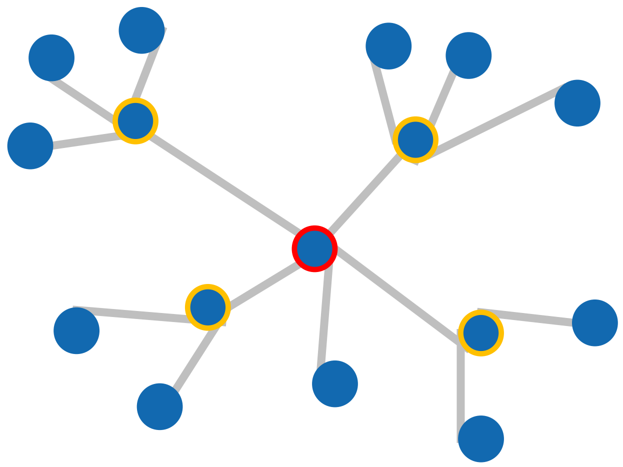
Nodes with relatively high degree

Setting #1 – Effect of heterogeneity in degree on differentiation

Nodes with relatively
high degree

● ●

High influx of migrants



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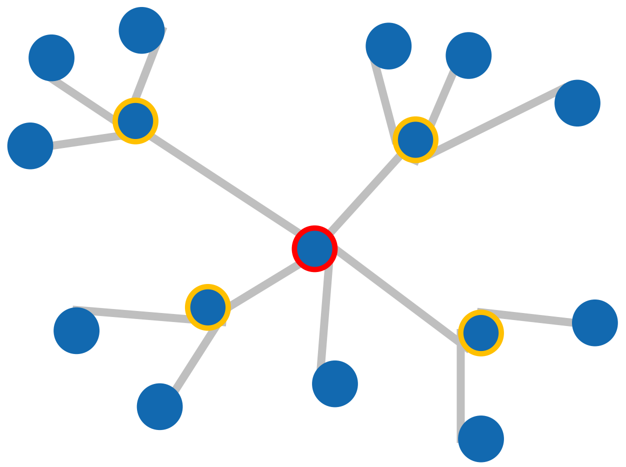
Increased competition



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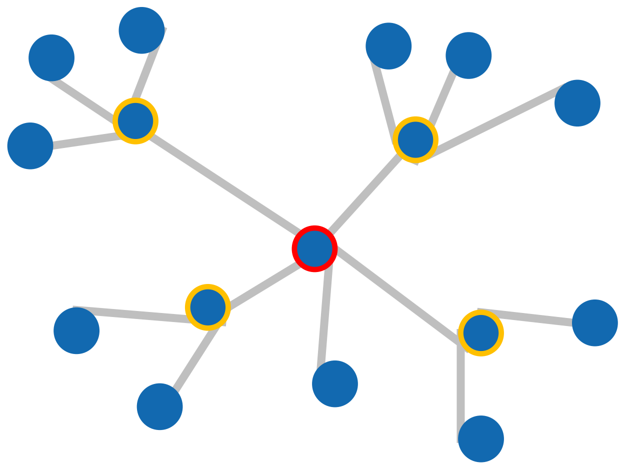

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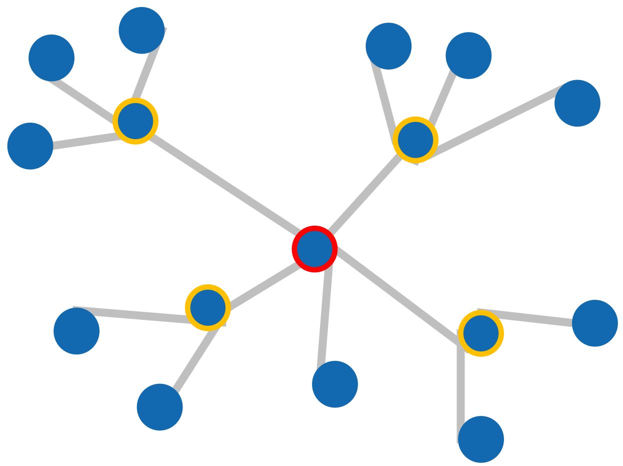

High influx of migrants

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Increased competition

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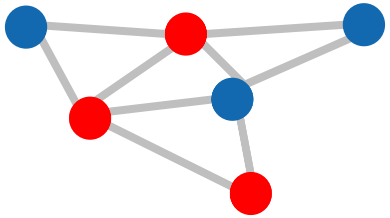
Higher death rate →

● ●

Bottlenecks
Barriers to dispersal

Setting #2 – Effect of topology & habitat heterogneity on differentiation



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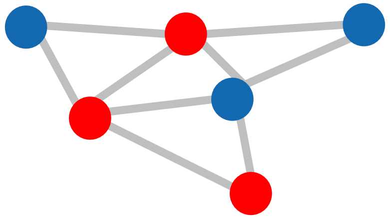

$$x_k = (u_k, s_k)$$

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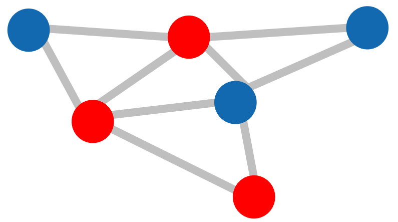

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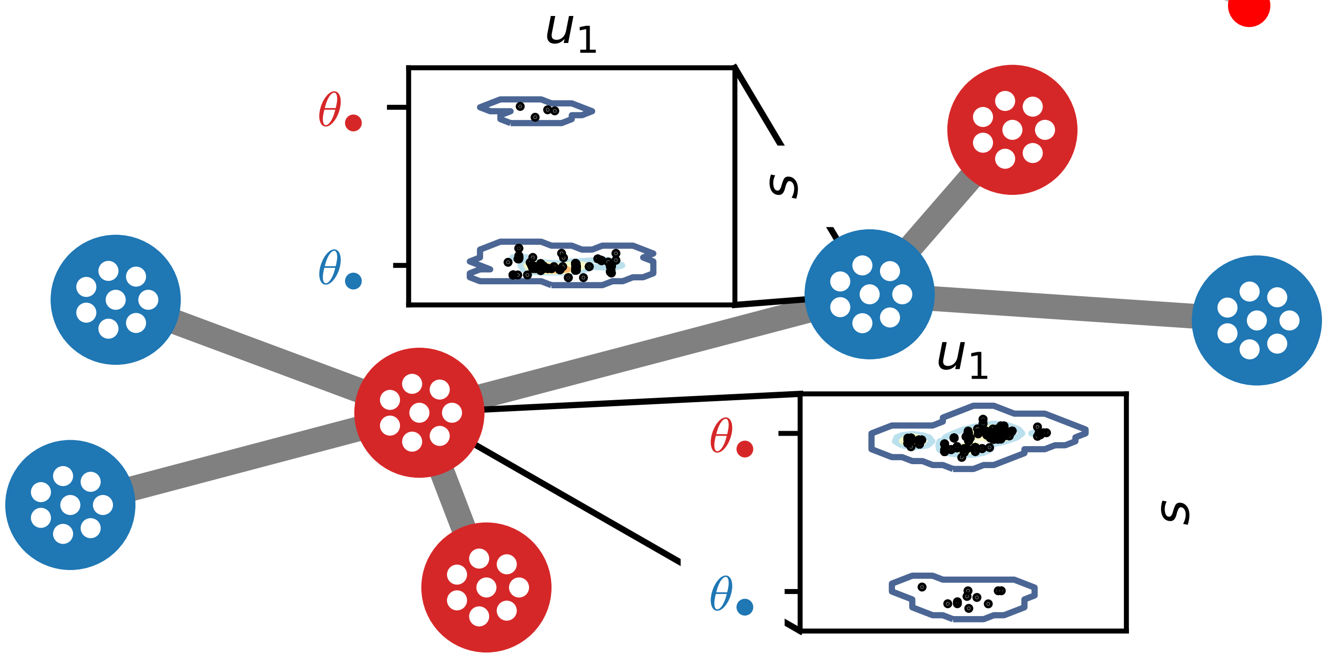

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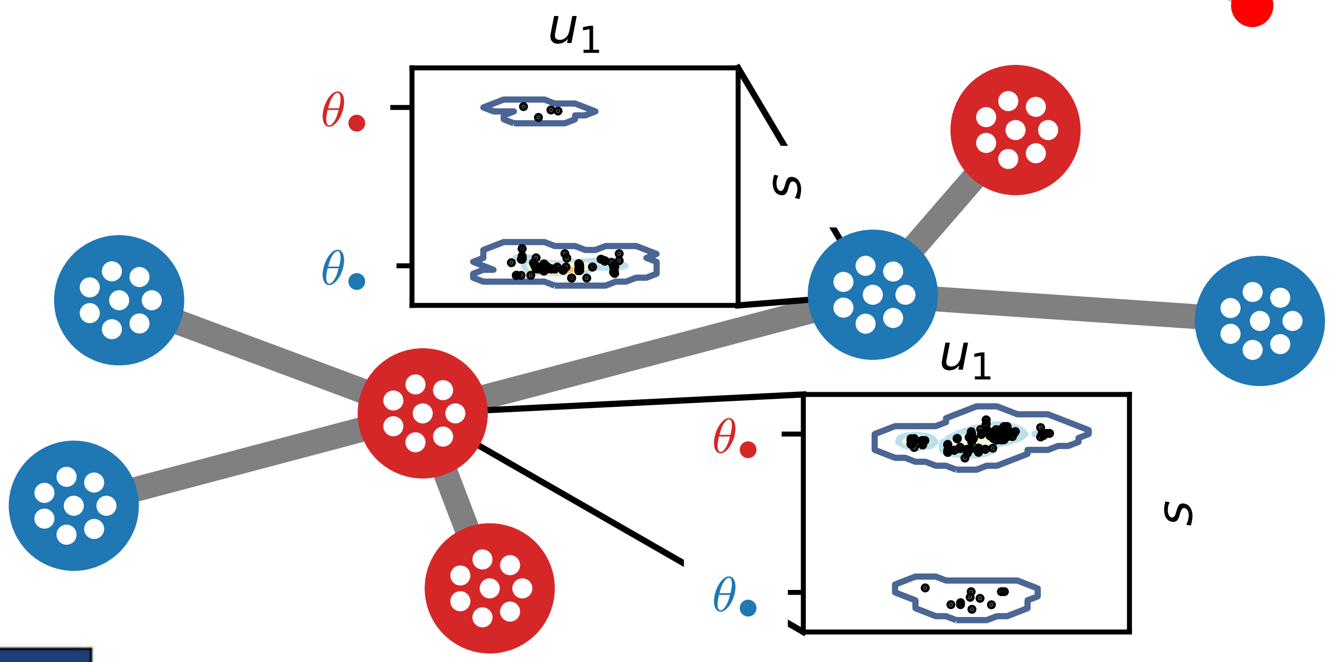
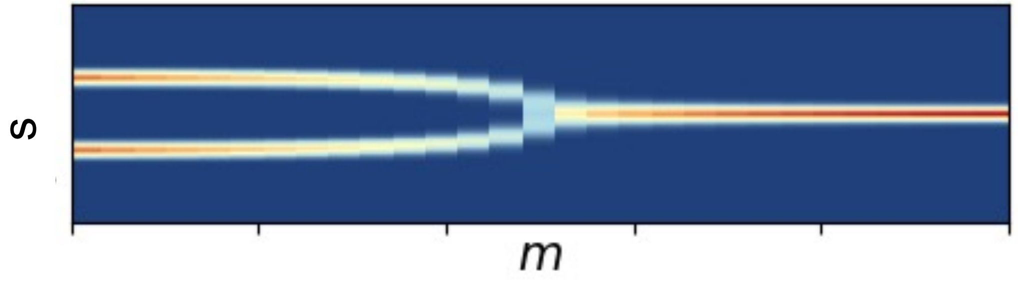
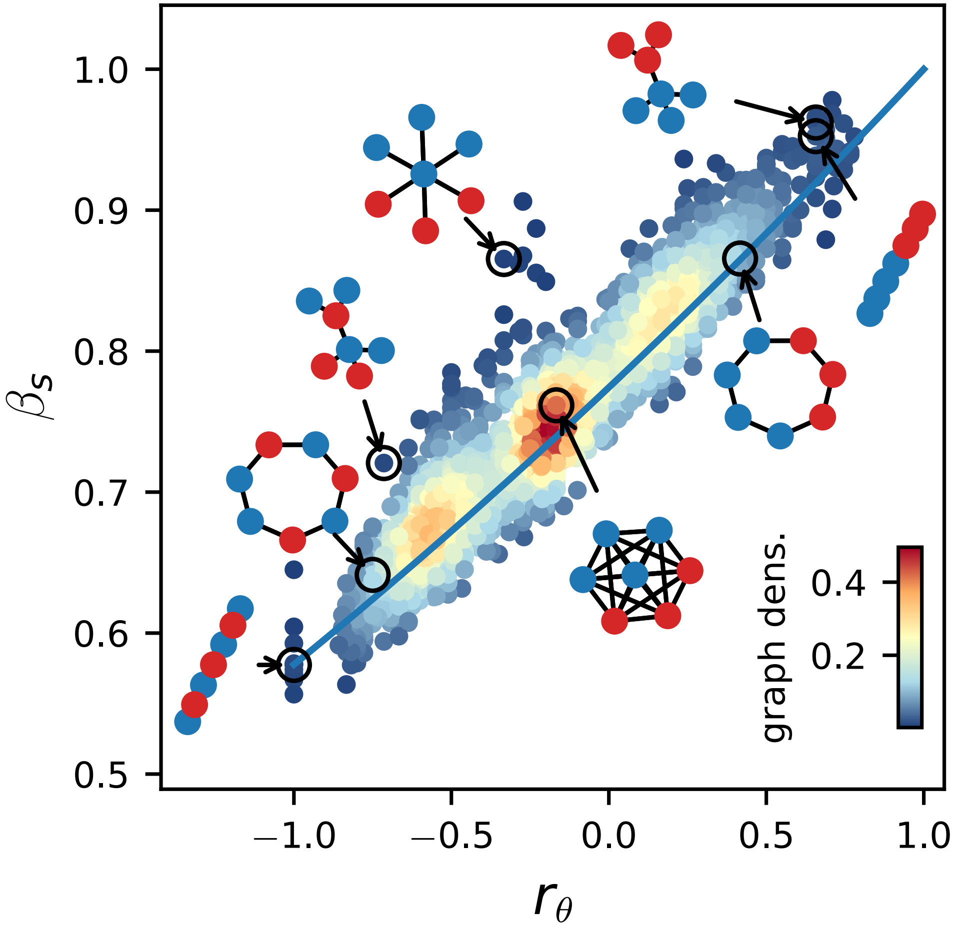
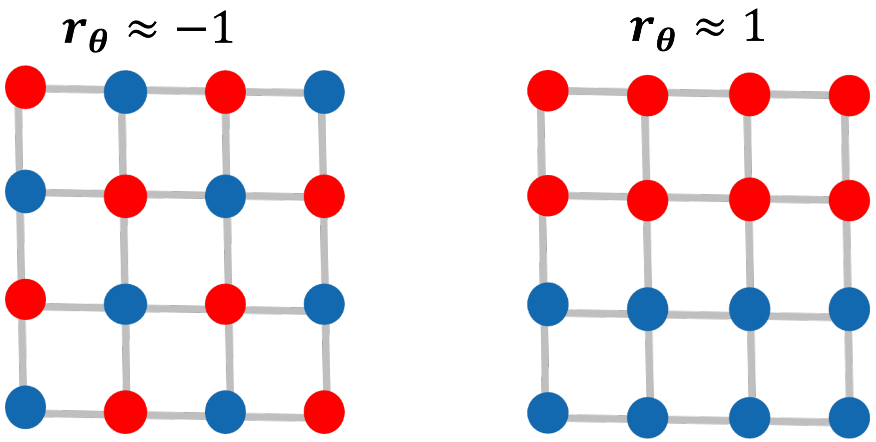




Setting #2 – Environmental assortativity r_θ drives differentiation through Isolation by Environment





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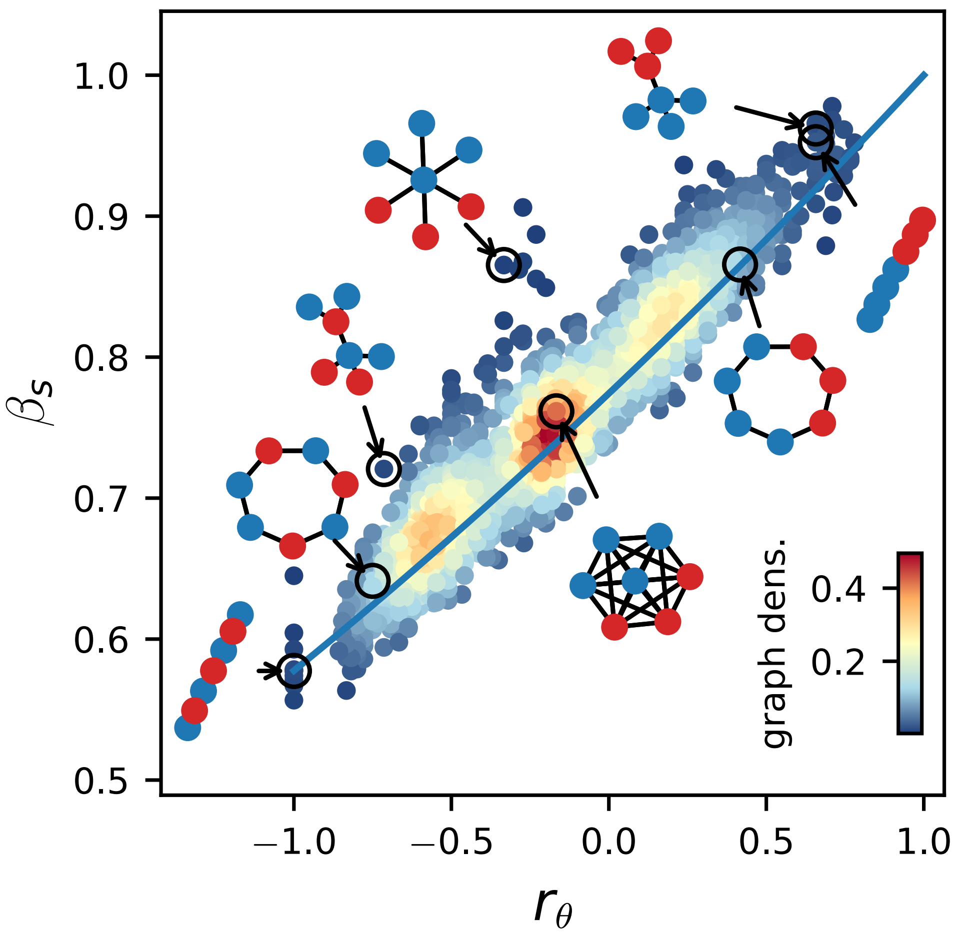

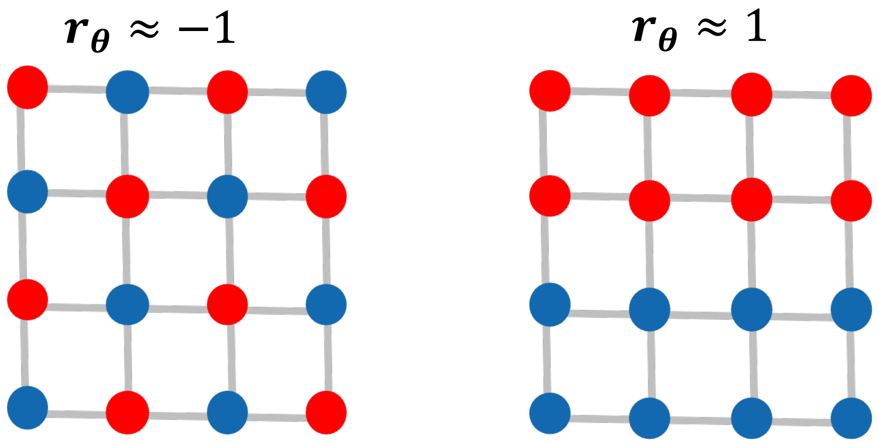

Mean field approach: *all vertices with a similar habitat are equivalent*

$$\partial_t \bar{n}_t^{\color{blue}\bullet}(s) = \bar{n}_t^{\color{blue}\bullet}(s) \left[b_{\color{blue}\bullet}(s)(1-m) - \frac{1}{K} \int_{\mathcal{S}} \bar{n}_t^{\color{blue}\bullet}(\mathbf{s}) d\mathbf{s} \right] + \tfrac{1}{2} \mu \sigma_\mu^2 (\Delta_s b_{\color{blue}\bullet} \bar{n}_t^{\color{blue}\bullet})(s)$$

$$+ \frac{m}{2} \left[(1 - r_\theta) b_{\color{red}\bullet}(s) \bar{n}_t^{\color{red}\bullet}(s) + (1 + r_\theta) b_{\color{blue}\bullet}(s) \bar{n}_t^{\color{blue}\bullet}(t) \right]$$

Summary

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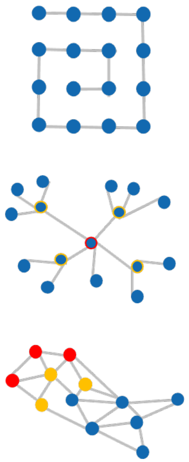

  - Characteristic length
  - Heterogeneity in degree
  - Environmental assortativity $\boldsymbol{r_\theta}$

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- **How do complex landscapes drive differentiation patterns?**

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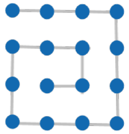
  - Characteristic length



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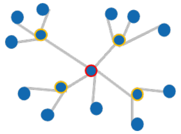
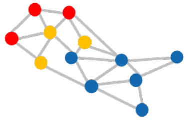


  - Environmental assortativity $\boldsymbol{r_\theta}$

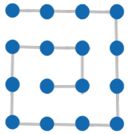


Boussange, V., & Pellissier, L. (2021). Topology and habitat assortativity drive neutral and adaptive diversification in spatial graphs. *BioRxiv*. https://doi.org/10.1101/2021.07.06.451404

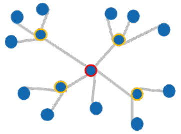
Summary

- **How do complex landscapes drive differentiation patterns?**

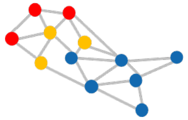
- Numerical and analytical results show that **three important graph properties control the level** of differentiation



  - Characteristic length



  - Heterogeneity in degree



  - Environmental assortativity $\boldsymbol{r_\theta}$

- **Theory validation**: using graph-based metrics

for realistic landscapes

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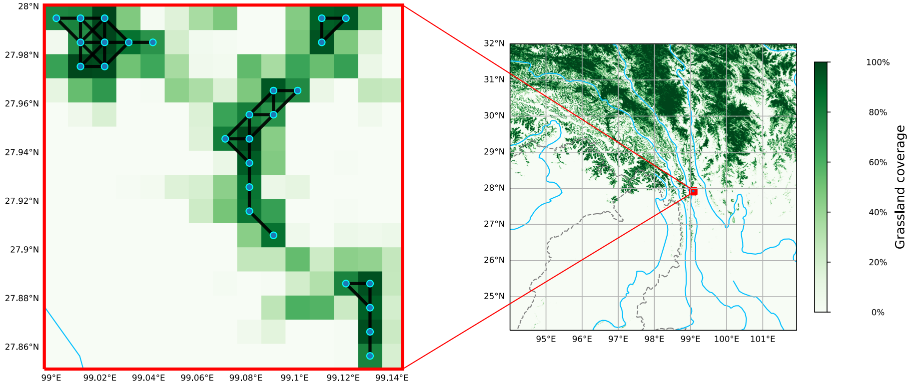

Acknowledgements

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Thanks!
(looking for a postdoc next year ☺)

Check out my personal website

to discover more
about my research